USING PUBLIC RECORDS TO SUPPORT THE PRODUCTIVE USE OF STUDENT MATHEMATICAL THINKING

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The more researchers understand the subtleties of teaching practices that productively use student thinking, the better we can support teachers to develop these teaching practices. In this paper, we report the results of an exploration into how secondary mathematics teachers' use of public records appeared to support or inhibit their efforts to conduct a sense-making discussion around a particular student contribution. We use cognitive load theory to frame two broad ways teachers used public records—manipulating and referencing—to support establishing and maintaining students' thinking as objects in sense-making discussions.

Keywords: Classroom Discourse; Instructional Activities and Practices.

Recommendations for mathematics teaching address the importance of using student thinking during whole class discussions as well as propose teaching practices that one may use to prepare for and facilitate whole class discussions (e.g., National Council of Mathematics Teachers, 2014). In our work, we study a particular teaching practice—a practice that takes advantage of instances of high leverage student thinking (Leatham et al., 2015) by making those instances of student thinking the object of whole-class sense making discussions (Van Zoest et al., 2016). As we studied teachers leading class discussions surrounding high leverage instances, we noticed variation in their use of public records of the initial student's thinking and subsequent discussion. This variation suggested that purposeful use of a public record—a physical representation that holds some degree of permanence and is visually accessible to all participants simultaneously—had the potential to play an important role in supporting the desired practice. The use of a public record as a support aligns with the tenets of cognitive load theory (Swellers, 1988; Swellers et al., 2011), which suggest that if the load for remembering other students' contributions during sense making discussions is lightened, then students can focus more on the sense-making actions they are being asked to carry out. A written representation of students' contributions can thus lighten this load. In this paper we report the results of our initial investigation into *how* teachers create and use public records of an initial student's mathematical contribution and subsequent discussion. These results provide insight into the ways public records can support teachers leading class discussions surrounding high leverage instances of student thinking.

Teachers' use of publicly accessible media (e.g., blackboards, white boards, etc.) is a common and widely accepted practice for sharing student contributions as well as mathematical content (Villareal & Borba, 2010). Additionally, some research has found that teachers' effective use of public records of student mathematical thinking corresponded with an increased level of

student mathematical activity during whole class discussions (Koehne et al., 2020). However, few studies have explicitly addressed how teachers use public records of student thinking during class mathematical discussions. Knowing *what* from a student's contribution gets captured and *how* it gets recorded is important since public records of student thinking provide permanence to student thinking, which can help maintain continuity during collaborative inquiry (Staples, 2007). Knowing more about how teachers use verbal and physical actions to reference and add to a public record can help researchers learn how teachers support students to engage with each other's ideas, a practice that has been linked to mathematical achievement (e.g., Webb et al., 2014). With respect to our research, knowing more about teachers' use of public records enables us to better understand how teachers effectively use student thinking during whole class sense making discussions. This understanding is important because there is evidence that teaching practices that use student thinking can be challenging to enact (e.g., Simpson & Haltiwanger, 2017; Peterson & Leatham, 2009). The more researchers understand the subtleties of teaching practices that productively use student thinking, such as creating and using public records, the better we can support teachers to develop these teaching practices.

Theoretical Framework

Our work focuses on understanding how mathematics teachers take advantage of teachable moments. We refer to these moments as **M**athematically significant pedagogical **O**pportunities to build on **S**tudent **T**hinking (MOSTs) (Leatham et al, 2015). Taking full advantage of MOSTs, what we refer to as the teaching practice of *building on a MOST* (or just *building*), is engaging the class in making sense of the MOST to better understand the mathematics of the MOST. As described elsewhere (Van Zoest et al., 2016), building consists of four elements:

- Establish: Make the MOST a clear object.
- Grapple Toss: Offer the MOST to the class with parameters that put them in a sense- making situation.
- Conduct: Conduct a whole-class discussion in which students collaboratively make sense of the MOST.
- Make Explicit: Facilitate the extraction and articulation of the mathematics of the MOST.

As can be seen from this description of building, the original student contribution—the MOST— needs to become a clear object and remain the object of consideration by the class throughout a sense-making discussion. Thus, in our work we focus on a *public record of a student's mathematical contribution and the class discussion of that contribution* (henceforth referred to as *public record*). Because of the central role of the student contribution to building, it seems reasonable to suppose that the way the public record of that contribution is created and used could help to facilitate (or possibly end up hindering) this discussion. One reason to suspect public records are an affordance when teachers are building comes from appealing to cognitive load theory.

Cognitive load theory (Swellers, 1988; Swellers et al., 2011) is a learning and instructional theory grounded on two types of memory —*long term* and *working*. Long term memory is the structure that serves as a person's permanent storage of information and is potentially unlimited in capacity. Working memory is the structure that processes incoming information in conjunction with information drawn from long-term memory. Working memory is very limited in capacity with regard to the amount of information processed and the duration that the information is held.

Given these limitations, learning and instruction impose *cognitive loads* of varying degrees on working memory. *Intrinsic load* is the load imposed by the basic structure of the information that is germane to learning. The other load is an *extraneous load* that is imposed by the instructional materials and activities in which the learner engages with the information. Since the extraneous load is not germane to learning and is typically under the control of a teacher, a goal of instructional design is to reduce the extraneous cognitive load so that a greater percentage of working memory resources can handle the intrinsic load. Extraneous load can occur when students try to hold information from one source while searching for and processing information from a separate source (Swellers et al., 2011). Teachers can help reduce the cognitive load by integrating the separate sources of information when the information needs to be considered simultaneously (Swellers et al., 2011). Additionally, researchers (e.g., Mousavi et al., 1995) have found that when a visual display is paired with speech during instruction, cognitive load can be reduced for students by drawing their attention to relevant pieces of the display that coordinate with referents in speech. Teacher use of public records has the potential to reduce extraneous load in such ways during whole-class sense making discussions, which would allow a greater percentage of working memory resources to be devoted to the intrinsic load imposed by the sense making activity.

In this paper, we address the following research question: How can teachers use public records of a MOST and of the subsequent discussion surrounding the MOST to support elements of the teaching practice of building on MOSTs?

Methods

This study is part of a larger project that included a data set of video-recordings of mathematics lessons from 6-12th grade teacher-researchers who endeavored to enact the teaching practice of building. The participants were selected because they expressed an interest in and a desire to know more about the productive use of student mathematical thinking. After receiving professional development related to building, the participants used mini-tasks designed to elicit anticipated MOSTs and then built on those MOSTs. For this study, we analyzed 27 lessons where a public record was used as part of the building practice.

We identified each time a public record was created in each lesson and then made note of whenever a change was made to the public record (e.g., an idea was added, circled, erased, etc). We refer to this creating and editing as *manipulating* the public record. After tracking how public records changed, we identified each instance of a teacher *referencing* the public record by identifying physical and verbal actions related to or directed at the public record. Physical actions include specific and general gestures that draw attention to parts of or the whole public record. Verbal actions include speech that draws attention to the public record in some way or implies the use or discussion of what was captured on the public record. Physical actions were captured using codes that determined whether gestures or looks were towards specific or general parts of the public record; codes for verbal actions included naming, pronoun, and repeat. Each enactment was coded individually by at least three researchers and then collaboratively reconciled.

Finally, we examined the collections of manipulating and referencing codes across enactments for each building practice element (Establish, Grapple Toss, Conduct, Make Explicit). Specifically, our analysis aimed at describing ways the teachers' use of public records appeared to support or inhibit the use of student thinking within each element of building. This study can thus best be characterized as theorizing grounded in data.

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Results

These results are organized by building element and describe the primary ways that teachers' manipulating or referencing of public records support the practice of building on MOSTs.

Using a Public Record to Support the Establish Element of Building

During the Establish element of building a teacher establishes (a) the precision of the MOST, ensuring that the MOST is clear enough for the class to engage with it; (b) the MOST as an object that can be identifiable throughout the sense making discussion; and (c) that the students have an intellectual need to engage with the MOST. (See Van Zoest et al., 2022 for further details about Establish and the other elements of building.) As described below, manipulating the public record has substantial potential to support establishing the precision of the MOST.

Manipulating the public record. In order to build on a MOST, a teacher first needs to create a foundation for a sense making discussion; creating a public record that clearly and completely captures the MOST helps to create this foundation. A teacher's honing of a student's contribution is important for creating a clear and complete public record. One form of honing is the teacher's use of symbols to represent connections students are making among mathematical ideas, such as drawing arrows that indicate a relationship among two entities rather than a written description of the relationship. Another form of honing involves a teacher capturing the essence of a student contribution while not recording it word-for-word, leaving off extraneous information or extra verbiage. Honing may help decrease cognitive load for students, as they would not have to attend to a word-for-word re-presentation of the student's contribution and try to carry out this honing themselves. Additionally, the public record needs to be created on the board space in a location that is visible for all to see, clearly separated from other information with room for additions, and that will not need to be removed as the discussion progresses since we want the MOST to be the object of the discussion. In each case, these manipulating actions support the work of establishing the MOST in preparation for the next element of building—Grapple Toss.

Referencing the public record. While referencing was not a primary use of the public record during Establish, our analysis did reveal some ways referencing may support this element of building. Teachers verbally referencing the public record with a name or label (e.g., this claim, this argument) contributes to making the MOST an object, because the name or label gives the MOST an identity that can be referenced in the public record throughout the discussion. Providing the MOST such an identity during the Establish element reduces the extraneous load for students during later elements of building because they need only remember the name or label and the location of the MOST in the public record as opposed to remembering the entire MOST throughout the discussion. Also, teachers often pointed at specific pieces of the public record as they wrote in order to confirm that what they were writing aligned with a student's thinking.

Using a Public Record to Support the Grapple Toss Element of Building

There are two critical aspects to the Grapple Toss element of building: a clearly established object that is offered to the class (the established MOST), and an action (on that object) that puts the students into a sense-making situation. As described below, referencing the public record has the potential to help students attend to both of these aspects.

Manipulating the public record. A major goal of manipulating the public record during the Establish element is to allow teachers to avoid most manipulation during the Grapple Toss element. That said, a subtle way manipulating can help emphasize the MOST as an object during Grapple Toss is a teacher circling, drawing a box, or underlining the MOST in the public record

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The teachers emphasizing the MOST in such a way can serve as a permanent gesture, similar to referencing.

Referencing the public record. A teacher's referencing of the public record during a Grapple Toss orients students to the details of the established object without the teacher having to repeat or revoice the MOST in its entirety. For example, a teacher could ask, "I want you to think about what Elisa has said. What do you find mathematically compelling or conflicting about this claim? *[pointing to a public record of the established MOST]*." Because the MOST is captured in the public record, the object can be referred to succinctly with "this claim" thus allowing the emphasis to be the sense making action. Referencing the public record in this way coordinates a visual mode of communication with the auditory mode of communication, thus potentially reducing the extraneous load of attending to the details of the MOST so that working memory resources can be devoted to the sense making action in the Grapple Toss question.

Using a Public Record to Support the Conduct Element of Building

After the established MOST has been tossed to the students, a teacher conducts a discussion that engages students in making sense of the MOST. As students offer additional contributions during this discussion, a teacher puts unrelated contributions aside, establishes contributions related to the MOST, and invites students to use related contributions to further their sense making of the MOST. As described below, both manipulating and referencing the public record can support these aspects of the Conduct element of building.

Manipulating the public record. As students offer their contributions during a discussion, the teacher may add other contributions related to the original contribution, the MOST, to the public record. When adding these related contributions, a teacher needs to attend to similar actions to those described in the Establish section above. In addition, teachers should consider the organization among contributions. This organization helps to support students in making connections between the new contribution and previous contributions. Parallelism, placement, and particularization are three important considerations for this public record organization. *Parallelism* (similar structuring) among the contributions may help the class make connections between those contributions. For example, the public record in Figure 1a has a variety of symbols and structure for the calculations that may lead to difficulties when comparing. By contrast, each student contribution in Figure 1b has been structured in the same way, potentially scaffolding student attention to their similarities and differences. The *placement* of additional contributions in relation to the original contribution can also support comparisons between contributions. For example, in Figure 1a it would likely take some effort for students to identify which of the contributions align with the MOST and which contributions do not. In Figure 1b, the contribution that agreed with the original MOST was vertically aligned below the MOST in a column on the left, while contradictory contributions were placed together in a column on the right. Teachers also may consider *particularizing* the contributions (e.g., drawing lines, using specific colors, assigning labels) to help distinguish the contributions. For example, in Figure 1a, it is difficult to identify when one contribution ends and another begins. However, in Figure 1b, each student's contribution is color coded differently, and each column of contributions is labeled with a "yes" or "no" so that students understand which contributions support or contradict the original. Also, within each contribution the initial and final prices are underlined to highlight the comparison each individual student's contribution is making. A well organized public record that addresses these considerations has the potential to reduce the extraneous cognitive load for students by integrating and aligning objects so that students' working memory resources can be allocated to processes germane to sense-making.

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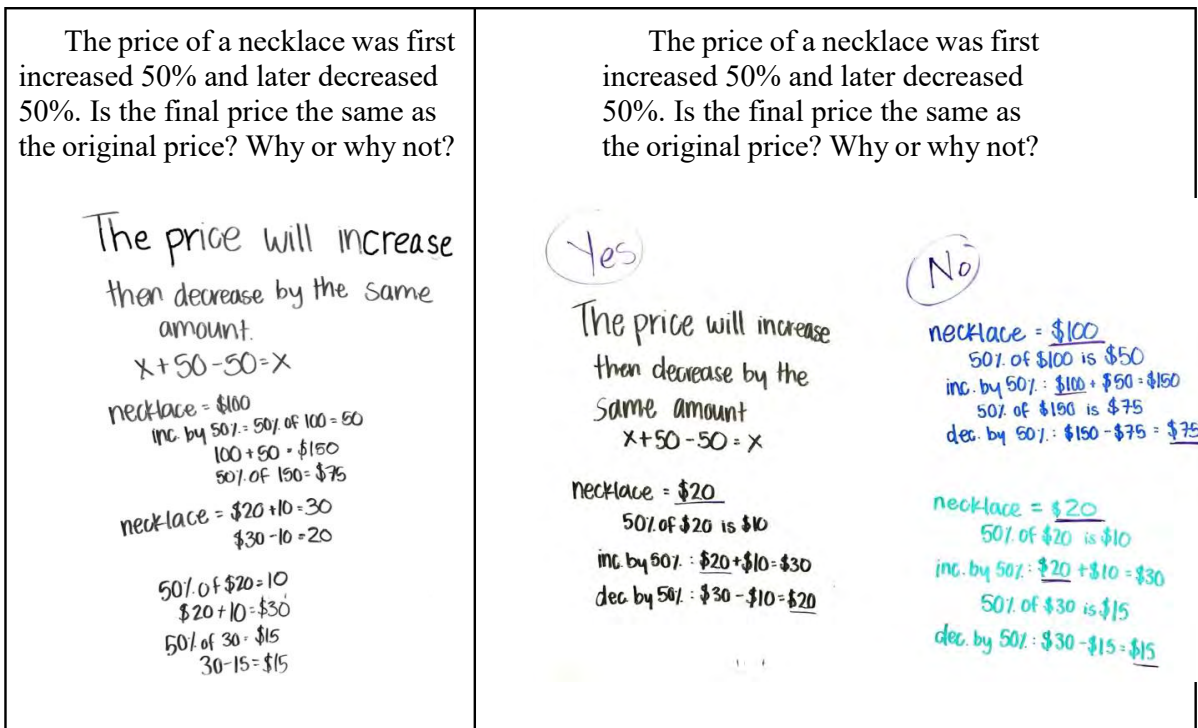| The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?	The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Figure 1: Public Record of MOST and Related Contributions During Conduct

Referencing the public record. One important part of conducting a sense-making discussion surrounding a MOST involves helping students to connect related student contributions (that have been established) to the MOST. Hence, a connecting move requires that both the MOST and a related student contribution be established as the object of the move. In addition, the move requires an action describing or requesting the nature of the connection that students are to make. Similar to referencing the public record during Grapple Toss, referencing to support connecting gives the action more prominence in the teacher's speech. However, because a teacher needs to reference two contributions in the public record, specific pointing gestures or verbal cues corresponding with the MOST and the related contribution become important, particularly because the public record may contain more information than these two contributions. For example, a teacher who asks, "How do you reconcile these two statements?" could use pointing gestures to make clear the objects in the public record that the class needs to reconcile. Differently, a teacher who asks, "How does this claim disprove Jaden's conjecture?" could use a pointing gesture to identify "this claim" in the public record, but the verbal naming "Jaden's conjecture" to identify the second contribution. This type of referencing reduces the cognitive load for students by clarifying the ambiguity of the referent for "this claim" and allowing the main focus to be on the requested reconcile action.

Another important part of using MOST related contributions is summarizing several established contributions so that they can be considered concurrently (i.e., synthesizing). A teacher's referencing supports the synthesis of student contributions to be connected by re-presenting the details of the contributions and helping students track those contributions in the public record. The public record provides permanence for student contributions so that teachers can use pointing gestures to support the class with attending to the details of multiple

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contributions. Supplementing speech with the use of the public record helps reduce the cognitive load of processing the sometimes lengthy details of contributions. Additionally, specific pointing gestures can clarify what piece of the public record corresponds with the teacher's speech. Such gestures reduce the cognitive load for students as they are directed to a specific piece of the public record and do not have to visually search for it on their own.

If a student shares a contribution during a discussion that is not related to the MOST, referencing the public record can help recenter the MOST as the focus of the discussion. In our research, we often found students responding to a Grapple Toss question with their own solutions to a task or their ways of thinking about an idea that was unrelated to or far from the focus of the MOST. Teachers can refocus students by gesturing at the MOST in the public record and telling students to focus on this idea, or referencing the MOST by its name while asking a student how their contribution is related to the MOST. Alternatively, teachers can use the same physical and verbal referencing of the MOST in the public record while reminding the class that the discussion is currently about making sense of the MOST.

Using the Public Record to Support the Make Explicit Element of Building

In the final element of building the teacher ensures that (a) the class agrees that the issue related to the MOST has been resolved, and (b) the mathematics of the MOST—the mathematical ideas that have emerged from making sense of the MOST—is explicitly articulated. We describe below how referencing can support resolving the MOST.

Manipulating the public record. In our data, teachers often did not manipulate the public record during Make Explicit since the ideas they wanted to discuss were already established, and connections and resolutions were made explicit throughout the discussion. However, we did see some manipulating that seemed to support Make Explicit. Editing the originally established MOST may further support students understanding the resolution of the MOST. Also, capturing the mathematics of the MOST succinctly may be important if the ideas will be used beyond the discussion of the MOST.

Referencing the public record. A teacher may reference the public record to emphasize the MOST as the object of a check-in question similar to the way referencing was used during Grapple Toss and Connect. For example, a teacher may ask, "Given our discussion for the past few minutes, how are you all thinking about our original argument? [gestures to the established MOST]." Another way a teacher may use referencing during Make Explicit is pointing to pieces of the public record that orient students to the details of ideas and connections said throughout the discussion that contribute to resolving the MOST.

Discussion and Conclusion

We reported the primary ways that a teacher's manipulating and referencing actions have the potential to support a teacher's productive use of student mathematical thinking when enacting the teaching practice of building on MOSTs. We begin the discussion by comparing and contrasting how manipulating and referencing actions play out across the four elements of building and conclude by reflecting on how these findings relate to the literature, particularly to cognitive load theory.

Although we described ways manipulating can support all elements of building, we see manipulating having a primary role during Establish and Conduct to provide permanence to student thinking shared in discussions. Manipulating that highlights pieces of the public record may occur during Grapple Toss or Make Explicit, but would be minimal as compared to altering contributions that would leave objects in the public record less clear for students. With respect to referencing, the combination and specificity of referencing actions often contributed to the extent

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to which these actions supported or hindered the building practice. Teachers clearly drew attention to particular contributions in the public record during Conduct with specific pointing gestures or using locator words (e.g., the upper right argument in blue). However, when teachers used vague pronouns (e.g., this calculation, that number) without pointing gestures to reference to a piece of a public record during conduct, the referent for the pronouns would likely be unknown for students.

There are at least two ways that teacher use of public records has the potential to reduce extraneous cognitive load imposed by split sources of information (Sweller et al., 2011) that inevitably occurs as a teacher builds on a MOST. First, sense making discussions require students to attend to a variety of information that is shared at different points in time and by different members of the class. Displaying (manipulating) the information in the public record can reduce the extraneous load imposed by having to remember the MOST and MOST-related contributions for the length of the discussion. Second, the public record may serve to reduce the extraneous load of attending to or searching for objects in the teacher's speech. During building, a teacher requests or positions students to engage in sense making actions with the MOST and MOST-related contributions. The teacher can reference these objects in the public record as opposed to revoicing or repeating them in their entirety, which has the potential to allow a greater percentage of working memory resources to be devoted to the load that is germane to the sense making of the mathematics of the MOST.

Our findings as to how manipulating and referencing can support elements of building extends the work of others investigating teacher use of student thinking to lead meaningful whole class discussions. Placement, parallelism, and particularization provide considerations for teachers manipulating and structuring public records that clearly and completely display student thinking during a sense-making discussion. These considerations offer insight as to how public records can be used to establish common ground, which is important for sustaining continuity of discussions on student thinking (Staples, 2007). During Grapple Toss, Conduct, and Make Explicit, referencing the public record can help support students' engagement with the details of each other's contributions. Referencing the public record in a Grapple Toss question or a request to connect ideas during Conduct can orient students to the specific objects with which they are to engage in sense making actions. This referencing provides insight into how teachers can use the public record as a resource to help students engage with the details of each other's thinking during whole class discussions, which can be key for students' mathematical learning (Webb et al., 2014).

Acknowledgments

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